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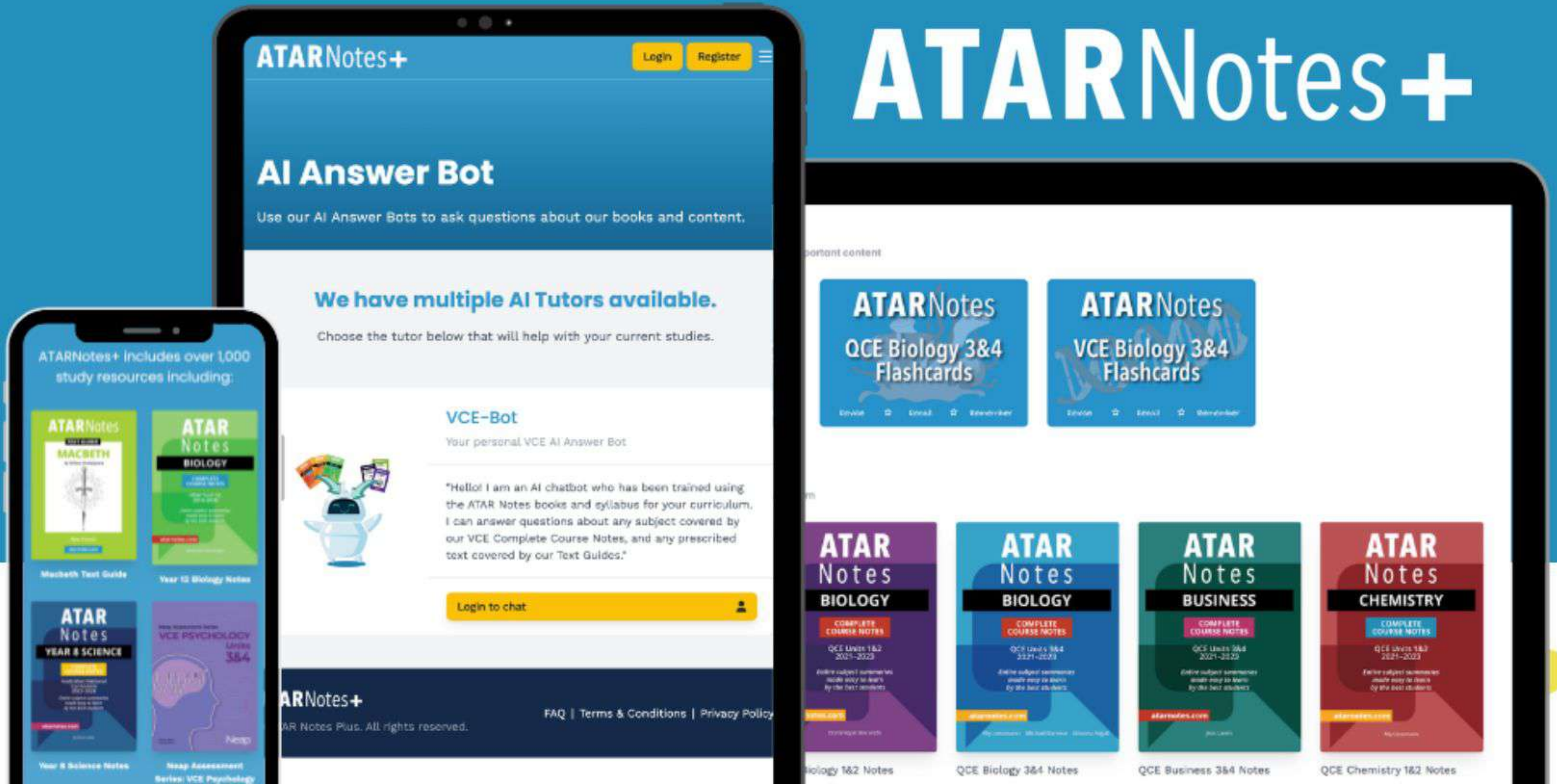

ATARNotes

Year 10 Maths

ATARNotes January Lecture Series

Presented by:
Michelle W

OVERVIEW

Topics to be covered

- **Content Block 1: Linear and Fundamental Equations**
  - Solving One-Step and Multi-Step Equations
  - Analysing Linear Relationships
- **Content Block 2: Quadratics**
  - Quadratic Expressions and Equations
  - Sketching Parabolas

Timing

- We'll run for 1.5 hours
- You're welcome to ask questions via the chat function ☺️

- **Fundamental Equations:**
  - **One-Step Equations**
  - **Multi-Step Equations**

- **Linear:**
  - **Gradients and Y-Intercepts**
  - **Parallel and Perpendicular Lines**

- **Fundamental Equations:**
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A **“one-step”** equation has: **One variable (unknown)**, several **known terms** and an **equals** sign.

Our goal is to **rearrange** the equation to **isolate** the **unknown**.

In “one-step” equations, we only need to perform **one** operation in order to do this.

Example: Solve for a:

$$a + 3 = 12$$

Step 1: To isolate a, we need to ‘*get rid of*’ the 3. We do this by subtracting 3 from each side of the equals sign, since it is currently being added. If you do something to one side of the equation, you must do the same to the other side:

$$a + \underbrace{3 - 3}_{} = \underbrace{12 - 3}_{}$$

Step 2: Evaluate

$$a + 0 = 9$$

Step 3: We don’t need to write the ‘0’:

$$a = 9$$

One-Step Equations: Operations

In the previous example, we solved the equation using **subtraction**.

Other **operations** we need to be familiar with are:

Addition, **multiplication** and **division**.

As well as:

Squaring and taking the **square root.**

Example: Addition: Solve for b:

$$b - 7 = 14$$

Step 1: To **isolate** b, we need to *'get rid of'* the 7. We do this by **adding 7 to each side** of the equals sign, since it is currently being **subtracted**. If you do something to one side of the equation, you must do the same to the other side:

$$b - 7 + 7 = 14 + 7$$

Step 2: Evaluate (remember you don't need to write the '0')

$$b = 21$$

Example: Multiplication: Solve for c:

$$\frac{c}{10} = -4$$

Step 1: To **isolate c**, we need to '*get rid of*' the division by 10. We do this by **multiplying each side of the equation by 10**, since it is currently being **divided** by 10. If you do something to one side of the equation, you must do the same to the other side:

$$\frac{c}{\cancel{10}} * \cancel{10} = -4 * 10$$

Step 2: Evaluate. The LHS will 'cancel' to leave us with the unknown.

$$c = -4 * 10$$

$$c = -40$$

Example: Division: Solve for d:

$$6d = -36$$

Step 1: To **isolate** d, we need to '*get rid of*' the multiplication by 6. We do this by **dividing each side of the equation by 6,** since it is currently being **multiplied by 6.** If you do something to one side of the equation, you must do the same to the other side:

$$\frac{\cancel{6}d}{\cancel{6}} = -\frac{36}{6}$$

Step 2: Evaluate. The LHS will 'cancel' to leave us with the unknown.

$$d = -\frac{36}{6}$$

$$d = -6$$

Solve the following for f:

$$\frac{f}{4} = 12$$

A: $\mathbf{3}$

B: $\mathbf{48}$

C: $\mathbf{8}$

D: $\mathbf{16}$

Solve the following for g:

$$g + \frac{1}{3} = -\frac{1}{2}$$

A: $-\frac{3}{2}$

B: -1

C: $-\frac{5}{6}$

D: $-\frac{1}{6}$

We've covered the operations: **Addition**, **Subtraction**, **Multiplication** and **Division**.

Now let's look at **squaring** and taking the **square root.**

Example: Squaring: Solve for h

$$\sqrt{h} = 3$$

Step 1: To **isolate** h, we need to '*get rid of*' the square root. We do this by **squaring each side of the equation.**

$$\sqrt{h}^2 = 3^2$$

Step 2: Evaluate. Squaring the square root leaves us with h by itself

$$h = 3^2$$
$$h = 9$$

Example: Taking the Square Root: Solve for k

$$k^2 = 25$$

The power of 2 suggests the equation might have two solutions. Here, they are 5 and -5.

Step 1: To **isolate k**, we need to *'get rid of'* the square (the power of 2). We do this by **taking the square root of each side of the equation.**

$$\sqrt{k^2} = \pm\sqrt{25}$$

Step 2: Evaluate. Similarly to the previous example, taking the square root of the square leaves us with **k** by itself.

$$k = \pm\sqrt{25}$$

$$k = \pm 5$$

The $\pm$ is very important! We don't know whether the answer is positive negative unless we are given more information.

Solve the following for m:

$$\sqrt{m} = \frac{1}{4}$$

A: $\frac{1}{16}$

B: $\frac{1}{2}$

C: $\frac{1}{8}$

D: $-\frac{1}{2}$

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  - **One-Step Equations**
  - **Multi-Step Equations**
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To solve **"multi-step"** equations, we need to perform **two or more operations** in order to isolate the unknown variable.

It's important to perform these operations in an **order** which is **efficient** and easy to follow.

Example: Solve for n

$$5n - 3 = 2$$

Step 1: Here, **n** is being **multiplied by 5**, and **3 is being subtracted** from that. We should start by "getting rid of" any *loose* terms: Add 3 to each side of the equation.

$$5n - 3 + 3 = 2 + 3$$

$$5n = 5$$

Step 2: Now we need to 'undo' the multiplication by 5. **Divide each side** of the equation by 5.

$$\frac{5n}{5} = \frac{5}{5} \longrightarrow n = \frac{5}{5} \longrightarrow n = 1$$

Example: Solve for p

$$3\sqrt{p} + 4 = 31$$

Step 1: Here, p is being **square rooted**, **multiplied by 3** and has **4 added** to it. Let's start by 'getting rid of' the *loose* term. **Subtract 4** from each side of the equation.

$$3\sqrt{p} + 4 - 4 = 31 - 4$$

$$3\sqrt{p} = 27$$

Step 2: Next we should get rid of the '3', we do this by **dividing by 3** on each side of the equation.

$$\frac{3\sqrt{p}}{3} = \frac{27}{3} \longrightarrow \sqrt{p} = \frac{27}{3} \longrightarrow \sqrt{p} = 9$$

Step 3: To get rid of the square root, **square both sides** of the equation.

$$\sqrt{p}^2 = 9^2 \longrightarrow p = 9^2 \longrightarrow p = 81$$

Solving Multi-Step Equations

Example: Solve for q

$$\frac{q^2}{4} - 14 = -10$$

Step 1: Again we will start by 'getting rid of' the ***loose*** term. **Add 14** to each side of the equation. Remember this cancels the 14 on the LHS.

$$\frac{q^2}{4} = -10 + 14$$

$$\frac{q^2}{4} = 4$$

Step 2: Multiply each side of the equation by 4. Remember: this cancels the 4 on the LHS.

$$q^2 = 4 * 4$$

$$q^2 = 16$$

Step 3: Take the square root of each side. Remember: This leaves q by itself.

$$q = \pm\sqrt{16}$$

$$q = \pm 4$$

Example: Solve for r

$$\frac{6-3r}{4}=-6$$

Step 1: Here we can't see any loose terms, everything is being divided by 4 on the LHS. We should undo this first by **multiplying each side of the equation by 4.** Remember: This will cancel the 4 on the LHS.

$$6-3r=-6*4$$
$$6-3r=-24$$

Step 2: Now we can see that 6 is a loose term. **Subtract 6** from each side of the equation.

$$6-6-3r=-24-6$$
$$-3r=-24-6$$
$$-3r=-30$$

Step 3: **Divide each side of the equation by -3.** Be careful with negative signs!

$$r=\frac{-30}{-3} \longrightarrow r=10$$

Solve the following for s:

$$7(2s + 8) = 56$$

A: $\mathbf{192}$

B: $\mathbf{0}$

C: $\mathbf{\frac{55}{2}}$

D: $\mathbf{8}$

Solving Multi-Step Equations: Your Turn

Solve the following for a:

$$\frac{2a^2 + 10}{6} = 7$$

A: 4

B: -4

C: $\frac{3}{2}$

D: Both A and B

E: None of the above

You will also need to know how to solve equations where the **unknown** appears on **both sides of the equals** sign.

We do this by **grouping like terms.**

Example: Solve for t

$$t + 7 = 3t + 5$$

Step 1: Group the like terms by putting the 't' terms together. Let's **subtract t from each side.**

This will 'get rid of' the t on the LHS, leaving us with **only one t term on the RHS.**

$$t - t + 7 = 3t - t + 5$$
$$7 = 3t - t + 5$$
$$7 = 2t + 5$$

Step 2: Now we have an equation that looks familiar! Get rid of the loose term by **subtracting 5 from each side.**

Continued: Example: Solve for t

$$t + 7 = 3t + 5$$

Step 2: Now we have an equation that looks familiar! Get rid of the loose term by subtracting 5 from each side.

$$7 = 2t + 5$$
$$7 - 5 = 2t + 5 - 5$$
$$2 = 2t$$

Step 3: Divide each side of the equation by 2 to get t by itself.

$$\frac{2}{2} = \frac{2t}{2}$$

$$1 = t$$

Step 4: Re-write to put the unknown on the LHS.

$$t = 1$$

Solving Multi-Step Equations

Example: Solve for u

$$3(u+5)=2u+3$$

Step 1: We should start by **expanding the bracket** on the LHS.

$$3u+15=2u+3$$

Step 2: Now let's **group the like terms**. **Subtract 2u from each side** so we have a single u term on the LHS.

$$3u-2u+15=2u-2u+3$$
$$3u-2u+15=3$$
$$u+15=3$$

Step 3: Finally, get rid of the 'loose' term. **Subtract 15 from each side** of the equation. Remember this leaves the u by itself.

$$u=3-15$$
$$u=-12$$

Solving Multi-Step Equations: Your Turn

Solve the following for v:

$$2(v-5)+3(v-7)=19v$$

A: $\frac{-31}{14}$

B: 10

C: $\frac{11}{19}$

D: $\frac{-11}{19}$

That's all for fundamental equations!

We went over:

-One-Step Equations

- Using the Operations: Addition, Subtraction, Multiplication and Division to isolate the pronumeral (unknown).
- Using the Operations: Squaring and Taking the Square Root to isolate the pronumeral.

-Multi-Step Equations

- Combining the above operations to isolate the pronumeral.
- Solving equations where the pronumeral appears on both sides of the equals sign by grouping like terms.

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Linear Equations

All linear equations can be written in the form:

$$y = mx + c$$

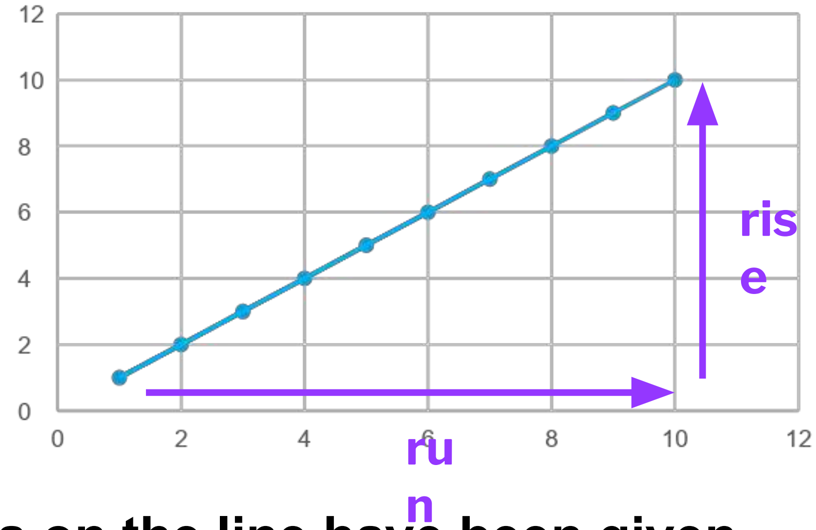
m: The '**gradient**' of the line
(How steep the line is)

c: The 'y-intercept' of the line
(Where the line crosses the vertical axis)

We can find the gradient of a line in a few ways, the two most common methods are:

-Using a graph to find the 'rise' and the 'run' of the line.

$$m = \frac{rise}{run}$$



-Using the formula for m when two points on the line have been given to you

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's practise the second method!

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the gradient of the line which goes through the points (1,3) and (5,6)

Step 1: We need to label the points given to us, using "x1" "y1" and "x2" "y2"

x1 y1 x2 y2

(1,3) (5,6)

Step 2: Substitute the correct numbers into the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{6 - 3}{5 - 1} \rightarrow m = \frac{3}{4}$$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Find the gradient of the line which passes through the points (3,4) and (2,10)

A: 6

B: -6

C: $\frac{-1}{6}$

D: $\frac{7}{2}$

Gradient: Your Turn

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Find the gradient of the line which passes through the points (10,-3) and (1,8)

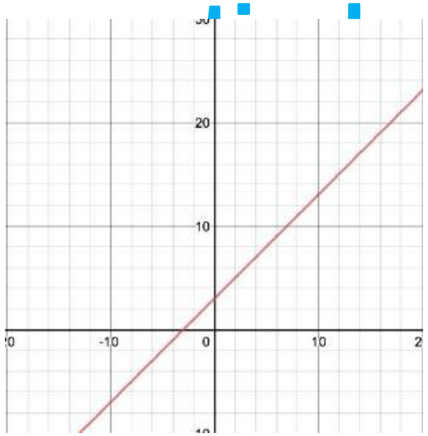
A: $\frac{5}{9}$

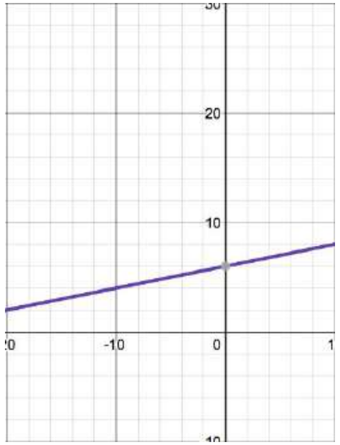
B: $\frac{5}{11}$

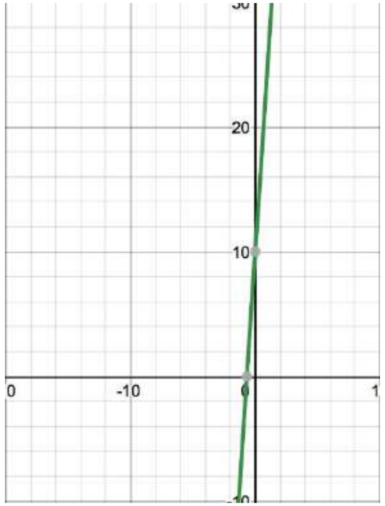
C: $\frac{-5}{9}$

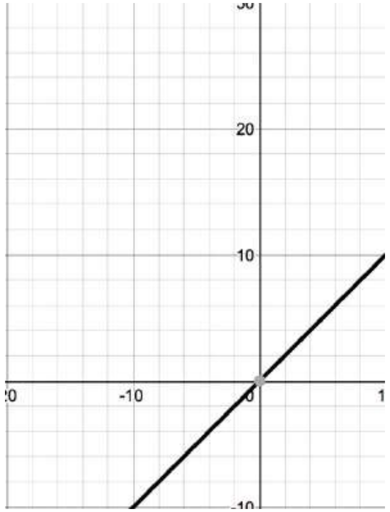
D: $-\frac{11}{9}$

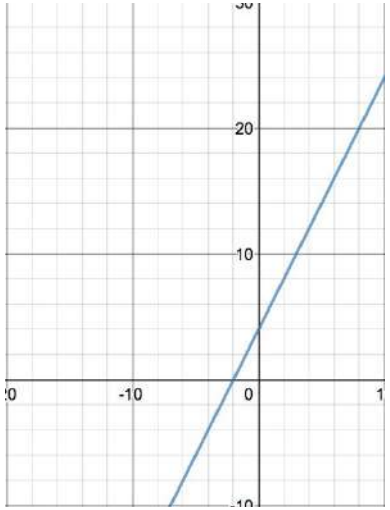
The y-intercept is where the line cuts the
xis!











**We can find the y-intercept in several ways. Today we will have
a look at three common methods!**

Method 1 for finding y-intercept:

Making $x = 0$ in any straight line equation

Method 2 for finding y-intercept:

Using the gradient and a given point to solve for c (y-intercept)

Method 3 for finding y-intercept:

Using two given points to solve for both m (gradient) and c (y-intercept)

Method 1 for finding the y-intercept

Example 1: If you are given the equation of a straight line in y = mx + c form, it is easy to identify c straight away, such as:

$$y = 3x + 4$$

Here's c

Here's m

Example 2: However, if the straight-line equation you are given is not in y = mx +c form, you can let x = 0 and solve for y.

$$3x + 14 = \frac{y}{2}$$

Step 1: Make x = 0 $\longrightarrow$ $3(0) + 14 = \frac{y}{2}$ $\longrightarrow$ $14 = \frac{y}{2}$

Step 2: Multiply each side by 2 to solve for y $\longrightarrow$ $28 = y$

This tells us that when x = 0, y = 28. So the y-intercept Is 28

What is the gradient, m, and the y-intercept, c, of the line with equation:

$$-x = 2y + 5$$

A: $m = -2 \quad c = -5$

B: $m = -\frac{1}{2}x \quad c = -\frac{5}{2}$

C: $m = 2 \quad c = 5$

D: $m = -\frac{1}{2} \quad c = -\frac{5}{2}$

Rearrange:

$$-x - 5 = 2y$$

$$-\frac{x}{2} - \frac{5}{2} = y$$

$$y = -\frac{x}{2} - \frac{5}{2}$$

m is the number in front of the x, NOT including the x!
c is the "loose" number on the end

Method 2 for finding the y-intercept

Example 1: You will often be told the gradient, m, of the line, and a co-ordinate point that it passes through. This is enough information to find the y-intercept!

Find the y-intercept of a line with a gradient of 7, passing through the point (2,6).

Step 1: Put the information you have into y = mx + c form. We know m = 7.

$$y = mx + c \longrightarrow y = 7x + c$$

Step 2: Now we use the co-ordinate point (2,6) to substitute x = 2 and y = 6

$$y = 7x + c \longrightarrow 6 = 7(2) + c$$

Step 3: Re-arrange to solve for c

$$6 = 7(2) + c \rightarrow 6 = 14 + c \rightarrow 6 - 14 = c \rightarrow -8 = c$$

This tells us that the y-intercept of the line is -8

What is the y-intercept of a line that has gradient 7 and passes Through the point (1,1)

A: 7

B: 4

C: -6

D: 8

Method 3 for finding the y-intercept

Example 1: Sometimes you will be given two points that the line passes through. You can find the equation of the line from these two points!

Find the equation of the line that passes through the points (2,14) and (3,19)

Step 1: Label the points using x1, y2, x2, y2 and then find the gradient,
'm' using the formula

x1 y1 x2 y2

(2,14) **(3,19)**

$$m = \frac{19 - 14}{3 - 2} \rightarrow m = \frac{5}{1} \rightarrow m = 5$$

Method 3 for finding the y-intercept

Example 1 (Continued)

So we know the line has gradient 5

Step 2: Put this information into y = mx + c form. We know m = 5.

$$y = mx + c \longrightarrow y = 5x + c$$

Step 3: Now we use the co-ordinate point (2,14) to substitute x = 2, y = 14

$$y = 5x + c \longrightarrow 14 = 5(2) + c$$

Step 4: Re-arrange to solve for c

$$14 = 5(2) + c \rightarrow 14 = 10 + c \rightarrow 14 - 10 = c \longrightarrow 4 = c$$

This tells us that the y-intercept of the line is **4, and we found the gradient to be 5. So we know that the equation of the line is:**

$$y = 5x + 4$$

What is the y-intercept of a line that passes through the points (-1,-4) and (2,6)

A: $-\frac{2}{3}$

B: $\frac{10}{3}$

C: 2

D: $\frac{1}{2}$

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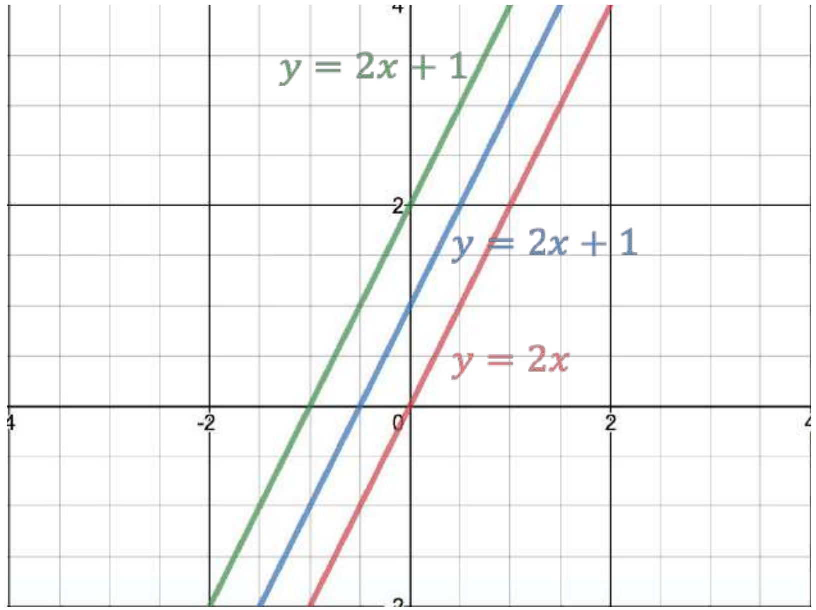
Parallel (of lines): Side by side and having the same distance continuously between them.

Lines which have the **same gradient** are always **parallel.**



These lines are parallel.

They all have a gradient m=2 with varying y-intercepts.

State an equation for a line which is parallel to:

$$y = \frac{7}{2}x + 4$$

First: What is the gradient of this line?

How many lines exist which are parallel to this line?

Parallel Lines

Example: What is the equation of the line which is parallel to the line $y = \frac{3}{2}x + 9$ and passes through the point (2,4)?

Step 1: Identify the gradient of the line we are given.

$$y = \frac{3}{2}x + 9 \qquad m = \frac{3}{2}$$

Step 2: Now we know that all lines which are parallel to $y = \frac{3}{2}x + 9$ have a gradient of 3/2.
Let's make a new line of the form y = mx + c and substitute our m value.

$$y = mx + c \Rightarrow y = \frac{3}{2}x + c$$

Step 3: We need to find the value of c such that our new line passes through (2,4). Substitute x=2 and y=4 and solve for c.

$$y = \frac{3}{2}x + c \Rightarrow 4 = \frac{3}{2}(2) + c \Rightarrow 4 = 3 + c \Rightarrow 1 = c$$

Continued: Example: What is the equation of the line which is parallel to the line $y = \frac{3}{2}x + 9$ and passes through the point (2,4)?

Step 4: We know that m=3/2, and we have found that c=1. We put this into y = mx + c form
and have found the equation of the line we were asked to find.

$$m = \frac{3}{2}, c = 1$$

$$y = mx + c \Rightarrow y = \frac{3}{2}x + 1$$

Parallel Lines: Your Turn

What is the equation of the line which is parallel to the line $y = \frac{-1}{6}x - \frac{4}{7}$ and passes through the point (4,2)?

A: $y = -\frac{1}{6}x$

B: $y = -\frac{4}{7}x + \frac{30}{7}$

C: $y = -\frac{1}{6}x + \frac{13}{3}$

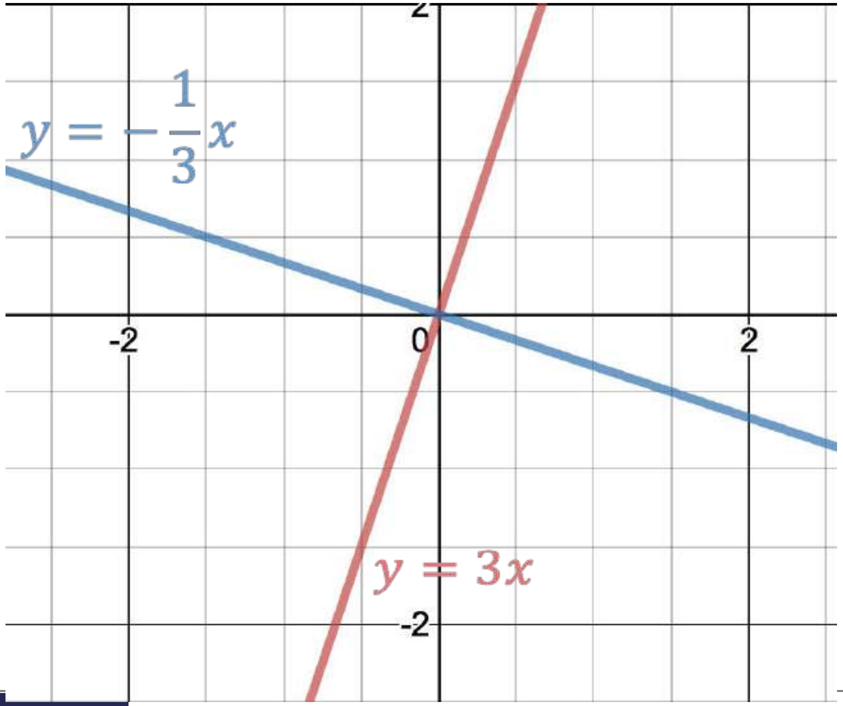
D: $y = -\frac{1}{6}x + \frac{8}{3}$

Perpendicular (of lines): Two lines which are perpendicular are at a 90º angle to each other.

Lines which are **perpendicular** to each other have gradients which are the **negative reciprocal** of each other.

These lines are perpendicular.

The angle between the lines at the point at which they intersect is 90º



Negative Reciprocal:

'Reciprocal' means to flip the number (the fraction).

To find the 'negative reciprocal' we flip the number and take the negative of it.

Examples:

The negative reciprocal of $\frac{2}{3}$

$$\frac{2}{3} \xrightarrow{\text{reciprocal}} \frac{3}{2} \xrightarrow{\text{negative reciprocal}} \frac{-3}{2}$$

The negative reciprocal of $\frac{-4}{5}$

$$\frac{-4}{5} \xrightarrow{\text{reciprocal}} \frac{-5}{4} \xrightarrow{\text{negative reciprocal}} \frac{- -5}{4} = \frac{5}{4}$$

The negative reciprocal of $\frac{1}{7}$

$$\frac{1}{7} \xrightarrow{\text{reciprocal}} \frac{7}{1} \xrightarrow{\text{negative reciprocal}} \frac{-7}{1} = -7$$

The negative reciprocal of -2

$$-2 \xrightarrow{\text{reciprocal}} \frac{-1}{2} \xrightarrow{\text{negative reciprocal}} \frac{- -1}{2} = \frac{1}{2}$$

Example: Give an equation of a line which is perpendicular to the line with equation $y = 8x + 16$

$$8 \xrightarrow{\text{reciprocal}} \frac{1}{8} \xrightarrow{\text{negative reciprocal}} \frac{-1}{8}$$

So **all** lines with **a gradient of -1/8** will be perpendicular.

$$y = -\frac{1}{8}x$$

$$y = -\frac{1}{8}x + 16$$

$$y = -\frac{1}{8}x - 27$$

$$y = -\frac{1}{8}x + 9042258257292$$

And infinitely many more…

Example: What is the equation of the line which is perpendicular to the line

$y = \frac{-9}{4}x + \frac{5}{3}$ and passes through the point (9,-3)?

Step 1: Identify the gradient of the line we are given.

$$y = \frac{-9}{4}x + \frac{5}{3} \quad\longrightarrow\quad m = \frac{-9}{4} \xrightarrow{\text{negative}} m = \frac{4}{9}$$

Step 2: Now we know that all lines which are perpendicular to $y = \frac{-9}{4}x + \frac{5}{3}$ have a gradient of 4/9. Let's make a new line of the form y = mx + c and substitute our m value.

$$y = mx + c \quad\Rightarrow\quad y = \frac{4}{9}x + c$$

Step 3: We need to find the value of c such that our new line passes through (9,-3). Substitute x=9 and y=-3 and solve for c.

$$y = \frac{4}{9}x + c \quad\Rightarrow\quad -3 = \frac{4}{9}(9) + c \quad\Rightarrow\quad -3 = 4 + c \quad\Rightarrow\quad -7 = c$$

Continued: Example: What is the equation of the line which is perpendicular to the line $y = \frac{-9}{4}x + \frac{5}{3}$ and passes through the point (9,-3)?

Step 4: We know that we need m=4/9, and we have found that c=-7. We put this into
y = mx + c form and we have found the equation of the line we were asked to find.

$$m = \frac{4}{9}, c = -7$$

$$y = mx + c \quad \Rightarrow \quad y = \frac{4}{9}x - 7$$

Perpendicular Lines: Your Turn

What is the equation of the line which is perpendicular to the line $y = \frac{-3}{13}x + \frac{3}{7}$ and passes through the point (2,8)?

A: $y = -\frac{3}{13}x + \frac{110}{13}$

B: $y = \frac{13}{3}x - \frac{2}{3}$

C: $y = -\frac{13}{3}x + \frac{50}{3}$

D: $y = \frac{13}{3}x - \frac{98}{3}$

Linear Equations

That's all for linear relationships!

We went over:

-Finding the gradient:

- using rise/run
- using the formula for m when given two points

-Finding the y-intercept of the line by:

- Letting $x = 0$ in any straight-line equation
- Rearranging for $y = mx + c$ form, then reading c straight off
- Finding c given m and a point
- Finding m, and then c, given two points

-Parallel and Perpendicular lines

- **Quadratic Expressions and Equations**
  - **Expanding**
  - **Factorising using Common Factors**
  - **Solving quadratic equations**

- **Sketching Parabolas**
  - **The General Shape**
  - **X and Y-Intercepts**

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All quadratic equations can be written in the form:

$$y = ax^2 + bx + c$$

If a = 1, we have a **"monic"** quadratic.

If a is any other number, we have
a **"non-monic"** quadratic.

Today we will introduce:

- Expanding and Factorising Quadratic Expressions
- Solving Quadratic Equations
- Tips for Sketching Quadratic Relationships

Expanding Quadratic Expressions

You might be familiar with **expanding single brackets:**

Examples:

$$2(x + 5) = 2 * x + 2 * 5$$
$$= 2x + 10$$

We often skip writing out the multiplication step

Examples:

$$3(x + 4) = 3x + 12$$

$$4(2x + 6) = 8x + 24$$

Now let's learn how to **expand two brackets** which are **multiplied together**.

For Example:

$$(x+1)(x+2)$$

$$= x * x + 2 * x + 1 * x + 1 * 2$$
$$= x^2 + 2x + x + 2$$
$$= x^2 + 3x + 2$$

We use a method called **'F.O.I.L'**.

F: First. Multiply the first terms from each bracket.

O: Outer. Multiply the 'outer' terms from each bracket.

I: Inner. Multiply the 'inner' terms from each bracket.

L: Last. Multiply the 'last' terms in each bracket.

Expanding Quadratic Expressions

Let's Practise!

$$(x + 3)(x + 5)$$

$$(x + 4)(x + 6)$$

$$(2x + 1)(x + 4)$$

Expanding Quadratics: Your Turn

The correct expansion of the following is:

$$(3x + 4)(x + 3)$$

A: $3x^2 + 7x + 12$

B: $3x^2 + 7x + 7$

C: $3x^2 + 13x + 12$

D: $x^2 + 7x + 12$

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We will learn one way of factorizing monic quadratics.

Method: Factorising into two brackets using common factors.

$$ax^2 + bx + c$$

Middle term

Final term

When factorising, you should always start by trying to find two numbers:

-which **ADD** together to give the middle term,

and

-**MULTIPLY** together to give the final term.

We want our final expression to be in the form:

$$(x + \_)(x + \_)$$

Method: Factorising into two brackets using common factors.

$$y = ax^2 + bx + c$$

Middle term

Final term

Let's look at some examples

Factorise:

$$x^2 + 7x + 12$$

The factors of 12 are:

1 and 12

2 and 6

3 and 4

This pair adds to 7!

We need two numbers:

-that **ADD to 7**, and

-**MULTIPLY to 12**

Now we know that we can write the expression as:

$$(x + 3)(x + 4)$$

Now Factorise:

$$x^2 + 9x + 20$$

We need two numbers:
-that **ADD to 9**, and
-**MULTIPLY to 20**

Which two numbers should we choose?

4 and 5

- If in doubt, list the factors of the "final" term until you find a pair which adds to the "middle" term

$$x^2 + 9x + 20 \longrightarrow (x+4)(x+5)$$

Factorise:

$$x^2 + 8x + 15$$

What do our two numbers need to do?

We need two numbers:
-that **ADD to 8**, and
-**MULTIPLY to 15**

→ **3 and 5**

So we can write the expression as:

$$(x + 3)(x + 5)$$

Factorise:

$$x^2 + 27x + 50$$

For more difficult expressions, list the factors of the final term

1 and 50
2 and 25
5 and 10

What do our two numbers need to do?

We need two numbers:
-that **ADD to 27**, and
-**MULTIPLY to 50**

$$(x + 2)(x + 25)$$

Factorise: $x^2 + 7x + 6$

A: $(x+5)(x+2)$

B: $(x+3)(x+4)$

C: $(x+6)(x+1)$

D: $(x+3)(x+2)$

The correct factorisation of the following is:

$$x^2 + 28x + 75$$

$$(x + 25)(x + 3)$$

Factorise:

$$x^2 - x - 6$$

We need to be careful when there are negative signs.

We still need two numbers:
-that **ADD to -1**, and
-**MULTIPLY to -6**

We look at factors of 6:
1 and 6
2 and 3

TIP: The **bigger number** of the pair *always* has the **same sign** (positive or negative) as the **middle term**! We have to figure out the sign of the smaller number ourselves.

So now we know that 3 must be negative. If we make 2 positive, we get:
-3 + 2 = -1 ✅
-3 x 2 = -6 ✅

(continued)

$$x^2 - x - 6$$

So now we know that 3 must be negative. If we make 2 positive, we get:

-3 + 2 = -1

-3 x 2 = -6

So our numbers are:

-3 and +2

$$x^2 - x - 6 \longrightarrow (x-3)(x+2)$$

Factorise:

$$x^2 + 6x - 16$$

Factors of 16:
1 and 16
2 and 8
4 and 4

We make the bigger number (8) positive because the middle term of the expression (6) is positive.

We need two numbers which:
-**ADD to 6**
-**Multiply to -16**

So **8 is positive**. We need to **make the 2 negative** so that we have:
8 – 2 = 6
8 x -2 = -16

Then we have $(x + 8)(x - 2)$

Which of the following is the correct factorisation of:

$$x^2 - 7x + 10$$

A: $(x-3)(x+4)$

B: $(x-5)(x+2)$

C: $(x+5)(x-2)$

D: $(x-5)(x-2)$

This year you will learn many other ways to **factorise monic** and **non-monic quadratics.**

Knowing how to factorise **using common factors** is **very important** and will help you to **solve problems** involving quadratics **quickly**!

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You will learn **many different ways** of solving quadratic equations during the year.

Today we will use our method of **factorising by common factors** in order to solve quadratic equations.

Solve for x:

$$x^2 + 4x + 3 = 0$$

First: Factorise the LHS into two brackets using the **common factors** method.

- We need two numbers that **ADD to 4**, and **MULTIPLY to 3.**
- 3 and 1

$$(x + 3)(x + 1) = 0$$

Second: Use the **'Null Factor Law'**

If two or more factors multiplied together equals zero. Then one or all of those factors is equal to zero.

Continued. Solve for x:

$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

Second: Use the **'Null Factor Law'**

If two or more factors multiplied together equals zero. Then one or all of those factors is equal to zero.

- So we **equate each of** the **two brackets on the LHS** to zero.

$$(x+3)(x+1) = 0$$

$$x + 3 = 0 \qquad x + 1 = 0$$

$$x = -3 \qquad x = -1$$

Continued. Solve for x:

$$x^2 + 4x + 3 = 0$$

$$(x + 3)(x + 1) = 0$$

$$x + 3 = 0$$
$$x = -3$$

$$x + 1 = 0$$
$$x = -1$$

So the solutions to the equation: $x^2 + 4x + 3 = 0$ are $x = -3$ and $x = -1$

Solve for x:

$$x^2 - 3x + 2 = 0$$

→ **Factorise the LHS into two brackets, then use “The Null Factor Law”**

-Two numbers that ADD to -3
-Multiply to 2

→ **-2 and -1**

$$(x-2)(x-1) = 0$$

Now use the **“Null Factor Law”:** If the product of any two expressions is zero, then one or both of the expressions is zero.

$$x - 2 = 0 \qquad x - 1 = 0$$

$$x = 2 \quad \textbf{and} \quad x = 1$$

Solve for x:

$$x^2 - 4x + 21 = 0$$

→ **Factorise the LHS into two brackets, then use "The Null Factor Law"**

-Two numbers that ADD to -4
-Multiply to 2

→ **-7and 3**

$$(x-7)(x+3) = 0$$

Now use the **"Null Factor Law":** If the product of any two expressions is zero, then one or both of the expressions is zero.

$$x - 7 = 0 \qquad x = 7$$

and

$$x + 3 = 0 \qquad x = -3$$

Solve for x

$$x^2 - 2x - 24 = 0$$

A: $x = -6\ x = 4$

B: $x = 2, x = 24$

C: $x = 3, x = 8$

D: $x = 6, x = -4$

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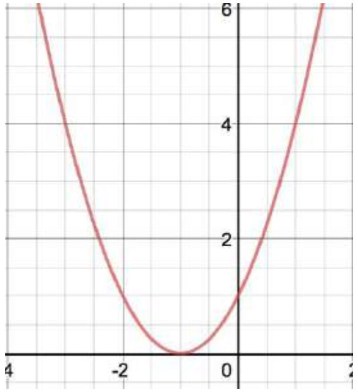
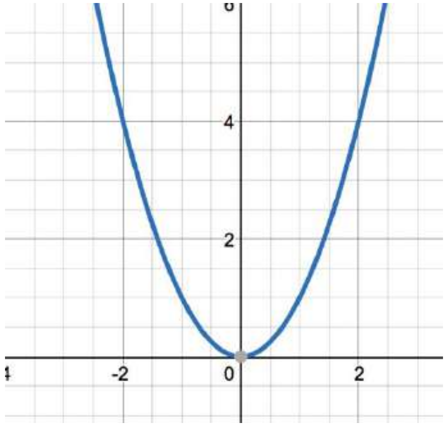
Sketching Parabolas: Shape

A 'parabola' is the u-shaped curve we obtain when we plot a quadratic equation, such as $y = x^2 + 2x + 1$



$y = x^2 + 2x + 1$

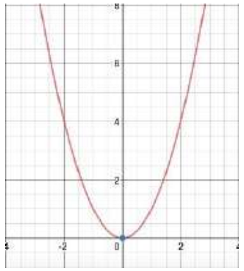
$y = x^2$



The 'standard' curve is given by the quadratic equation $y = x^2$

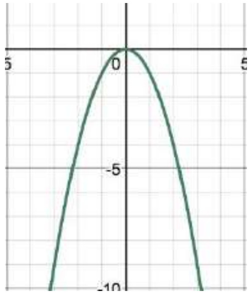
Tips for Sketching Quadratic Relationships

$$ax^2 + bx + c$$



Check to see if 'a' is positive or negative:

If a is positive:
The parabola is a **happy face** 🙂!



If a is negative:
The parabola is a **sad face** ☹!

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Finding the x and y-intercepts of a Parabola

- Find the y-intercept by letting x=0
- Find the x-intercept(s) by letting y=0

→ This will involve solving a quadratic equation!

- Sketch a smooth “u” shaped curve through these intercepts

Let's practice!

Sketch:

$$y = x^2 - 4x - 5$$

Step 1) Find the y-intercept by making x=0

$$y = 0^2 - 4(0) - 5 \rightarrow y = -5$$

Step 2) Find the x-intercept(s) by making y=0

$$0 = x^2 - 4x - 5$$

(Now factorise the RHS and use the null-factor law)

What does the RHS factorise to?

Let's practice!

Sketch:

$$y = x^2 - 4x - 5$$

Step 2) Find the x-intercept(s) by making y=0

$$0 = x^2 - 4x - 5$$

(Now factorise the RHS and use the null-factor law)

What does the RHS factorise to?

- Two numbers that ADD to -4 and MULTIPLY to -5
- -5 and 1

$$y = (x - 5)(x + 1)$$

Let's practice!

$$y = x^2 - 4x - 5$$

Sketch:

Step 2) Find the x-intercept(s) by making y=0

$$0 = (x-5)(x+1)$$

(Null Factor Law)

$$x - 5 = 0 \quad and \quad x + 1 = 0$$

$$\boldsymbol{x = 5 \quad and \; x = -1}$$

These are our x-intercepts!

Let's practice!

$$y = x^2 - 4x - 5$$

Sketch:

Step 3) Draw a smooth curve through the y and x-intercepts

We found that:

The y-intercept was at y = -5 (x=0) → (0,-5)

The x-intercepts were at x = 5 and x = -1 (y=0) → (5,0) and (-1,0)

Tips for Sketching Quadratic Relationships

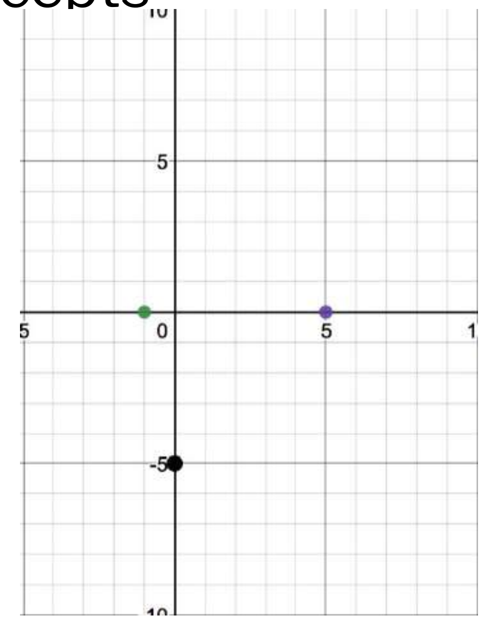
Let's practice!

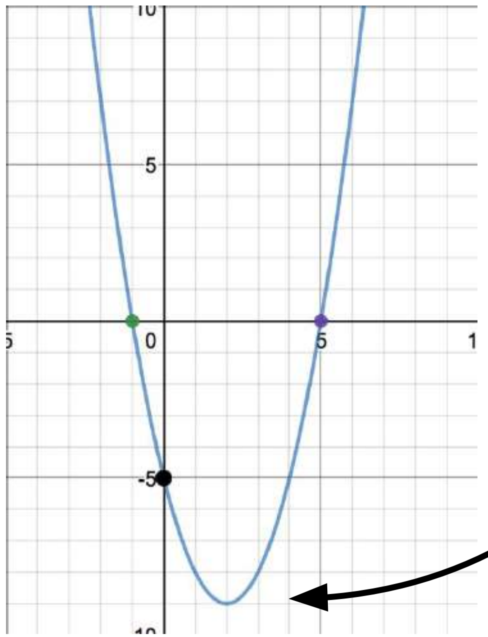
No negative sign so it's a happy face!

$$y = x^2 - 4x \ - 5$$

Sketch:

Step 3) Draw a smooth curve through the y and x-intercepts





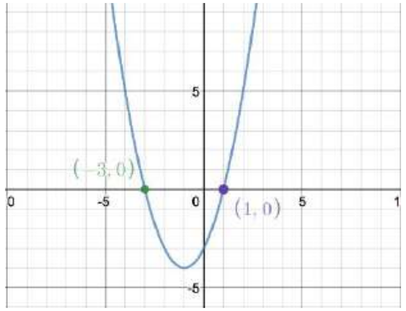
Make sure the turning point is halfway between x-intercepts

Your turn!

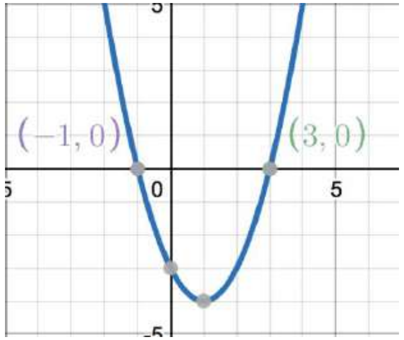
Which of the following could be the graph of:

$$y = -(x+3)(x-1)$$

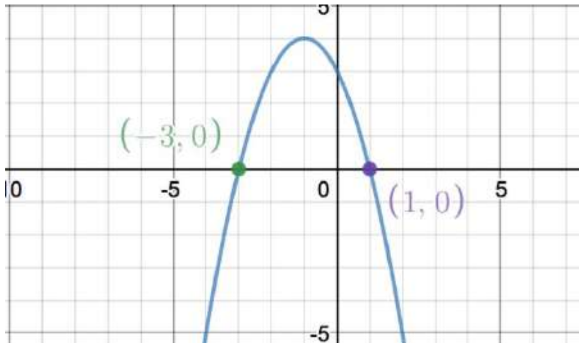
A:



B:



C:



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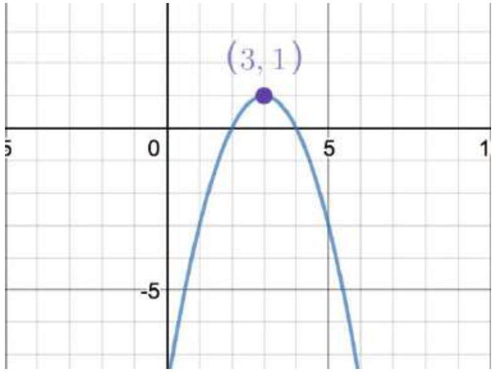

That’s all for quadratics!

We went over:

-Factorising Quadratic Expressions

- By looking at the equation then using two brackets (monic
- By splitting the middle-term
- By completing the square

-Solving Quadratic Equations

- By factorising then using the Null Factor Law
- By using the quadratic formula

-Sketching Parabolas

- From the x and y intercepts
- From the turning point form

Thank you for coming!
Any more questions?